RMS Electron Density Fluctuation at 1 AU

A. L. Berman

TDA Engineering Office

Analytic expressions at 1 AU for the average RMS Electron Density Fluctuation and the ratio of RMS Electron Density Fluctuation to Electron Density, both as functions of the observational time scale, are constructed from average spacecraft in situ density measurements at approximately 1 AU and columnar phase fluctuation measurements over a wide variety of signal closest approach points. Additionally, the (one-dimensional) Electron Density Fluctuation spectrum and the Doppler phase fluctuation "scale" are derived, and various extrapolations to the region interior to 1 AU are made.

I. Introduction

Solar Wind modeling is essential for predicting the effects of the Solar Wind upon spacecraft telecommunications, particularly for those spacecraft in various phases of Solar Conjunction. Fundamental to the process of modeling the Solar Wind is a description of the time-scale-dependent Electron Density Fluctuation. There now exist substantial in situ measurements of electron density (as a function of time) at approximately 1 AU and columnar phase fluctuation measurements (also as a function of time) at a wide variety of signal closest approach points, the two of which can be reconciled and subsequently synthesized to construct "average" analytic expressions to describe Electron Density Fluctuation. In pursuit of this goal, in situ Electron Density measurements by the Mariner 5 and Vela 3 spacecraft will be combined with Viking and Helios Doppler Phase Fluctuation observations.

II. The Ratio of RMS Electron Density Fluctuation to Mean Electron Density

It is convenient to start with a description of the relevant parameters as in Ref. 1:

$N_e(r,t)$ = instantaneous electron density, electrons/cm^3

r = radial distance

t = time

such that the mean electron density becomes:

$$N_e(r) = \frac{1}{t} \int_0^t N_e(r,t)dt$$

and the RMS electron density fluctuation is hence:

$$n(r) = \left[\frac{1}{t} \int_0^t \{N_e(r,t) - N_e(r)\}^2 dt \right]^{1/2}$$

The ratio of "local" electron density fluctuation to mean electron density is simply defined as:

$$\epsilon \equiv \frac{n(r)}{N_e(r)}$$

As was pointed out in Ref. 1, the most frequent abuse of the parameter ϵ is that it is treated as a constant; in fact, it must be treated as a function of the time-scale over which the RMS value is computed:

$$\tau_n = \text{RMS time-scale}$$

$$\epsilon = \epsilon(\tau_n)$$

In the following section, the time-scale-dependent form of ϵ will be obtained at 1 AU.

III. The Time-Scale-Dependent Form of ϵ at 1 AU

The work of Goldstein and Sisco (Ref. 2) shows that the in situ (one-dimensional) power spectrum of electron density begins to fall off after one solar rotation, or approximately 2.4×10^6 seconds. For time-averaging periods longer than this, one would expect the value of ϵ to be nearly constant. To compute a long-term value of ϵ, one can use the following:

(1) Mariner 5 data (5 months) (Ref. 2)

$$N_e \,(1 \text{ AU}) = 9.2 \text{ electrons/cm}^3$$

$$n \,(1 \text{ AU}) = 5.6 \text{ electrons/cm}^3$$

$$\epsilon \,(1 \text{ AU}) = 0.609$$

(2) Vela 3 data (2 years) (Ref. 3)

$$N_e \,(1 \text{ AU}) = 7.7 \text{ electrons/cm}^3$$

$$n \,(1 \text{ AU}) = 4.6 \text{ electrons/cm}^3$$

$$\epsilon \,(1 \text{ AU}) = 0.597$$

The following value of ϵ at 1 AU will hence be adopted:

$$\epsilon(\tau_n) = 0.6$$

$$\tau_n > 2.4 \times 10^6 \text{ seconds}$$

One now desires the value of ϵ for time-scales less than one solar rotation. The form of the temporal columnar fluctuation spectrum (P) is well known to be power law with fluctuation frequency (ν):

$$P(\nu) \propto \nu^{-K_0}$$

The average value of K_0 from Helios and Viking Doppler phase fluctuations (Ref. 4) is [1]:

$$K_0 = 2.42$$

One knows (Cronyn, Ref. 5) that if a temporal columnar fluctuation spectrum is of the form:

$$P(\nu) \propto \nu^{-K_0}$$

then the equivalent in situ fluctuation spectrum (P_n) will be:

$$P_n(\nu) \propto \nu^{-K_0 + 1}$$

hence, one writes for the in situ density fluctuation spectrum:

$$P_n(\nu) \propto \nu^{-1.42}$$

The relationship between fluctuation frequency and time-scale is:

$$\nu \propto \tau_n^{-1}$$

and the relationship between the RMS density fluctuation and the density fluctuation spectrum is therefore:

$$\left[n(\tau_n) \right]^2 = \int P_n(\nu) \, d\nu$$

$$\propto \int \nu^{-1.42} \, d\nu$$

$$\propto \nu^{-0.42}$$

$$\propto \tau_n^{0.42}$$

Finally, one arrives at:

$$n(\tau_n) = K_1 \tau_n^{0.21}$$

for the electron density fluctuation dependence upon τ_n at 1 AU. Assuming a constant value of N_e at 1 AU and applying

---

[1] Coincidentally, $k_0 = 2.42$ is the exact average of the six experiments detailed in Table I of Ref. 4.

the value of $\epsilon = 0.6$ at $\tau_n = 2.4 \times 10^6$ seconds, one would have for $\epsilon(\tau_n)$:

$$\epsilon\,(\tau_n)\;=\;0.6\left(\frac{\tau_n}{2.4\times10^6}\right)^{0.21} \qquad \tau_n < 2.4\times10^6 \text{ seconds}$$

$$\epsilon\,(\tau_n)\;=\;0.6 \qquad\qquad\qquad\quad \tau_n > 2.4\times10^6 \text{ seconds}$$

Using the average value of N_e at 1 AU (Ref. 6):

$$N_e\,(1\text{ AU}) \simeq 7.5 \text{ electrons/cm}^3$$

one would have for the electron density fluctuation at 1 AU:

$$n\,(\tau_n)\;=\;4.5\left(\frac{\tau_n}{2.4\times10^6}\right)^{0.21} \text{ electrons/cm}^3$$

$$\tau_n < 2.4\times10^6 \text{ seconds}$$

$$n(\tau_n) = 4.5 \text{ electrons/cm}^3$$

$$\tau_n > 2.4\times10^6 \text{ seconds}$$

For example, considering the time-scale applicable to 60-second sample interval Doppler noise (15 samples), one has:

$$\tau_n = 900 \text{ seconds}$$

$$n(900) = 0.86 \text{ electrons/cm}^3$$

$$\epsilon(900) = 0.11$$

By way of comparison to actual spacecraft (in situ) results, one has for a 10^4-second ($\sim$2.8 hours) time scale:

$$\epsilon(10^4) = 0.19$$

This is compared in Fig. 1 to typical in situ spacecraft electron density measurements over 10^4 seconds.

Combining the ideas presented in this section with the following nominal electron density model (Ref. 7):

$$N_e(r) = \frac{2.39\times10^8}{r^6} + \frac{1.67\times10^6}{r^{2.30}}$$

$$r = \text{heliocentric distance, solar radii}$$

one can obtain the following radially dependent electron density fluctuation:

$$n(\tau_n) = 0.6\left(\frac{\tau_n}{2.4\times10^6}\right)^{0.21}\left\{\frac{2.39\times10^8}{r^6}\right.$$

$$\left. + \frac{1.67\times10^6}{r^{2.30}}\right\} \text{ electrons/cm}^3$$

$$\tau_n < 2.4\times10^6 \text{ seconds}$$

$$n(\tau_n) = 0.6\left\{\frac{2.39\times10^8}{r^6} + \frac{1.67\times10^6}{r^{2.30}}\right\} \text{ electrons/cm}^3$$

$$\tau_n > 2.4\times10^6 \text{ seconds}$$

and additionally, the following radially dependent (one-dimensional) electron density fluctuation spectrum ($P_n(\nu)$):

$$P_n(\nu) = 3.2\times10^{-4}\nu^{-1.42}\left\{\frac{2.39\times10^8}{r^6}\right.$$

$$\left. + \frac{1.67\times10^6}{r^{2.30}}\right\}^2 \text{ cm}^{-6}\ \text{Hz}^{-1}$$

$$\nu > (2.4\times10^6)^{-1}$$

$$P_n(\nu) = 0$$

$$\nu < (2.4\times10^6)^{-1}$$

$$\nu = \text{fluctuation frequency, Hz} = \tau_n^{-1}$$

This average spectrum is compared to the Mariner 5 results of Ref. 2 in Fig. 2.

IV. The Scale of Doppler Phase Fluctuation

In Ref. 6, the scale for Doppler phase fluctuation is given as:

$$L(a) = \frac{0.43}{\epsilon^2}\left(\frac{a}{r_0}\right)\left(\frac{\tau}{60}\right)^{1.4}, \text{ km}$$

where

L = scale, km

a = signal closest approach point

r_0 = solar radius

τ = Doppler sample interval ($\tau_n = 15 \times \tau$), seconds

Substitution for ϵ yields:

$$L(a) = 100\left(\frac{a}{r_0}\right)\left(\frac{\tau}{60}\right), \text{km} \qquad \tau_n < 2.4 \times 10^6 \text{ seconds}$$

and

$$L \propto \tau \qquad \tau_n < 2.4 \times 10^6 \text{ seconds}$$

and hence the Doppler phase fluctuation scale is seen to be:

$$L \propto a$$

$$L \neq f(\tau) \qquad \tau_n > 2.4 \times 10^6 \text{ seconds}$$

References

1. Berman, A. L., "Proportionality Between Doppler Noise and Integrated Signal Path Electron Density Validated by Differenced S-X Range," in *The Deep Space Network Progress Report 42-38*, Jet Propulsion Laboratory, Pasadena, California, April 15, 1977.

2. Goldstein, B., and Sisco, G. L., "Spectra and Cross Spectra of Solar Wind Parameters From Mariner 5," in *Solar Wind*, edited by Sonett, C. P.; Coleman, P. J., Jr.; and Wilcox, J. M., National Aeronautics and Space Administration, Washington, D.C., 1972.

3. Hundhausen, A. J., Bame, S. J., Asbridge, J. R., and Sydoriak, S. J., "Solar Wind Proton Properties: Vela 3 Observations from July 1965 to June 1967," *Journal of Geophysical Research, Space Physics*, Vol. 75, No. 25, September 1, 1970.

4. Berman, A. L., "Phase Fluctuation Spectra – New Radio Science Information to Become Available in the DSN Tracking System, Mark III-77, in *The Deep Space Network Progress Report 42-40*, Jet Propulsion Laboratory, Pasadena, California, August 15, 1977.

5. Cronyn, W. M., "Density Fluctuations In the Interplanetary Plasma: Agreement Between Space Probe and Radio Scattering Observations," *Astrophysical Journal*, Vol. 171, February 1, 1972.

6. Berman, A. L., "Electron Density in the Extended Corona: Two Views," in *The Deep Space Network Progress Report 42-41*, Jet Propulsion Laboratory, Pasadena, California, October 15, 1977.

7. Berman, A. L., and Wackley, J. A., "Viking S-Band Doppler RMS Phase Fluctuations Used to Calibrate the Mean 1976 Equatorial Corona," in *The Deep Space Network Progress Report 42-38*, Jet Propulsion Laboratory, Pasadena, California, April 15, 1977.

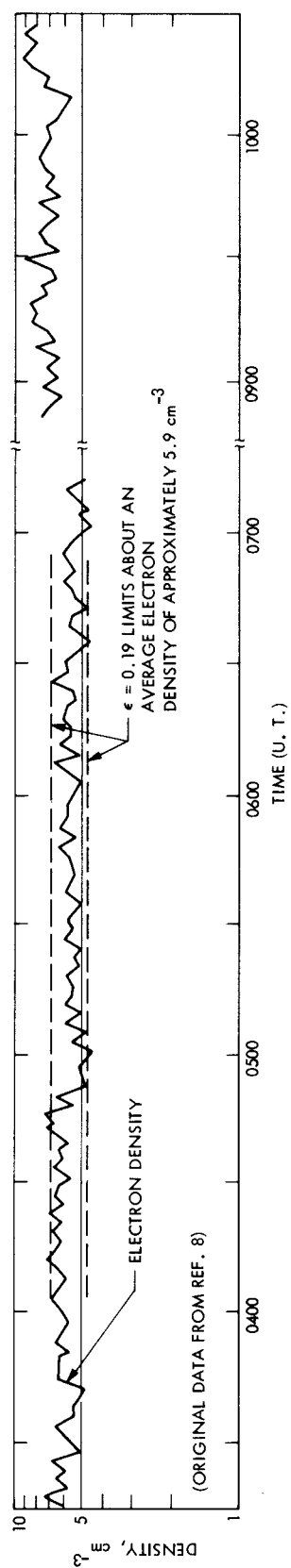8. Brandt, J. C., *Introduction to the Solar Wind*, W. H. Freeman and Company, San Francisco, 1970.

Fig. 1. Comparison of ratio of electron density fluctuation to electron density (ε) to actual spacecraft in situ electron density measurements for $\tau_n = 10^4$ seconds
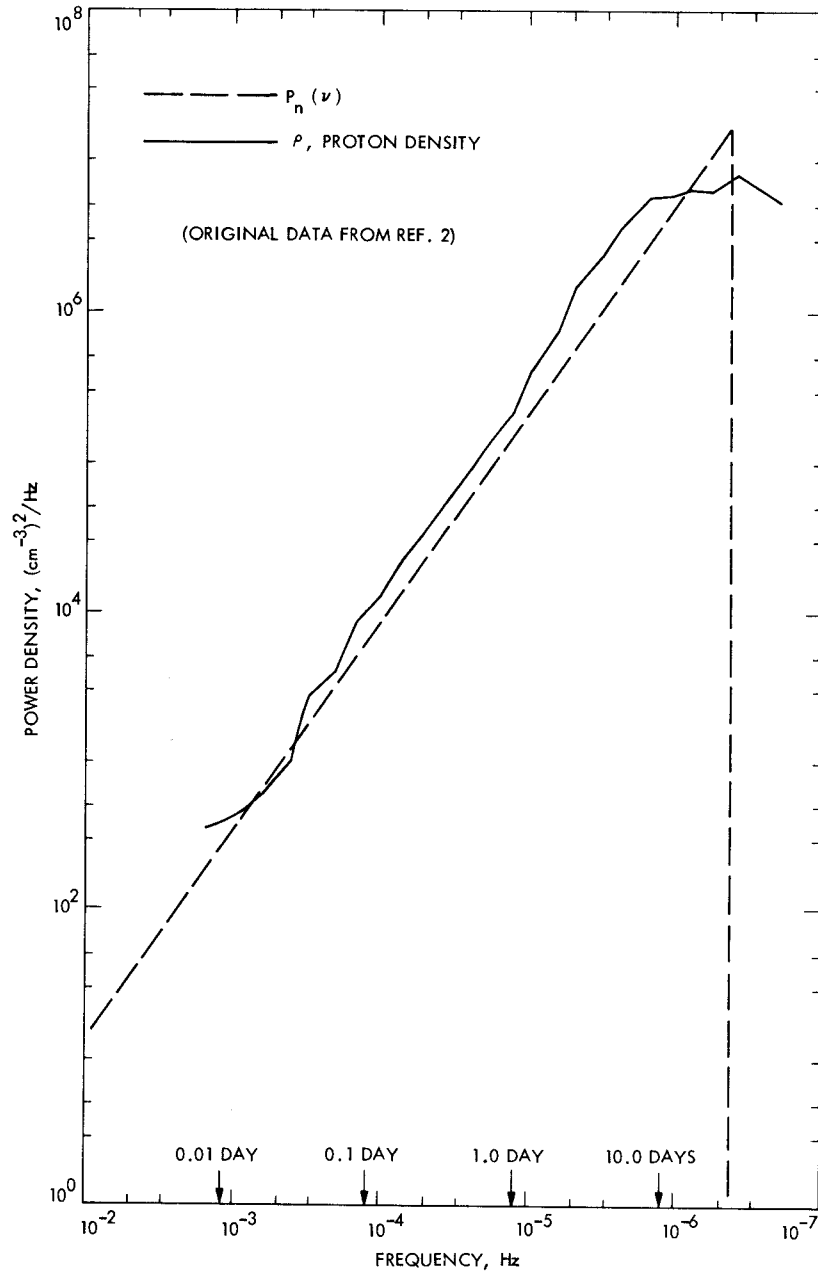
Fig. 2. Comparison of the (one-dimensional) electron density fluctuation spectrum at 1 AU to the actual Mariner 5 proton density spectrum at approximately 1 AU